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TRACKING MODEL OF AN ADAPTIVE LATTICE FILTER FOR A LINEAR CHIRP SIGNAL IN NOISE

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ABSTRACT

This paper studies the behavior of the (PARCOR) (PARtial CORrelation) coefficients of the stochastic gradient adaptive lattice filter in response to a complex linear chirp FM signal in white Gaussian noise. A single-stage model for the behavior of the coefficients is developed, and the recovery error is derived. Also, an accurate model for a two-stage filter is derived when either the chirp rate or the input noise is increased. The second stage PARCOR coefficient is derived assuming that the first coefficient has reached its steady state. The accuracy of the model is verified by computer simulation.

1. INTRODUCTION

The performance of adaptive filters for sinusoids in white Gaussian noise has been studied in [1],[2] for the LMS FIR filter and [3] for the Lattice Filter. The tracking characteristics for a complex chirp signal have been investigated in [4],[5],[6],[7] for the LMS and RLS Filters. In this paper, we will study the behavior of the PARCOR (PARtial CORrelation) coefficients and the recovery error of the SG (Stochastic Gradient) Adaptive Lattice Filter in response to a complex linear chirp signal in white Gaussian noise. The expected values of the optimal PARCOR coefficients and a first-order, single-stage analytical model of the expected values of the PARCOR coefficients based on the SG update algorithm are derived and compared with simulations. It is noted that the optimal model contains only the chirp sinusoid component at iteration k. However simulation shows that both the chirp sinusoid component at iteration k and at iteration 1 are present. This is the "shadow" effect observed in [8]. The single-stage analytical model will be used to explain how the update algorithm of the SG Lattice retains this "shadow" component. A simple single-stage model of the recovery error, which measures the ability of the filter to extract the signal from the input will also be presented.

A detailed two-stage model is derived for the cases of noiseless and noisy input signals. It was shown in [5] that even though the RLS algorithm exhibits superior convergence characteristics over the LMS algorithm, its tracking performance is in fact inferior to that of LMS. This is because convergence and tracking are different phenomena. The results derived in this paper show that this is also the case for the SG Adaptive Lattice Filter, where the convergence and tracking rates are defined by independent parameters. Furthermore, the analysis shows that the PARCOR coefficients are reduced from their optimal values when either the chirp rate or the input noise is increased. The second stage PARCOR coefficient is derived assuming that the first coefficient has reached its steady state. The accuracy of the model is verified by computer simulation.

2. THE STOCHASTIC GRADIENT LATTICE FILTER

The lattice filter structure to be considered is shown in Figure 1. The lattice order recursion equations are given by

$$e_f(k|n) = e_f(k|n-1) - K_n^f e_b(k-1|n-1) \quad (1)$$

$$e_b(k|n) = e_b(k-1|n-1) - K_n^b e_f(k|n-1) \quad (2)$$

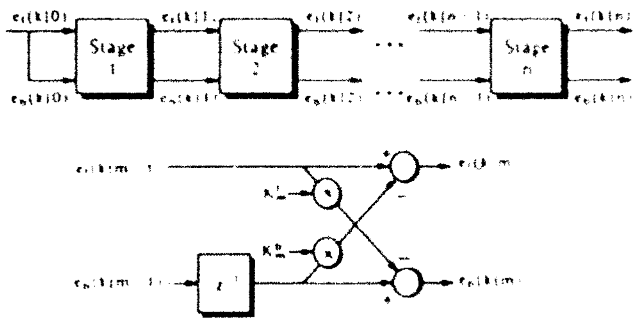

Figure 1 Lattice Filter Structure

The value of $K_n^f(k)$ that minimizes the mean squared forward prediction error $E[e_f^2(k|n)]$ is given by

$$K_n^{opt}(k) = \frac{E[e_f(k|n-1)e_b^*(k-1|n-1)]}{E[e_b^2(k-1|n-1)]} \quad (3)$$

where we have assumed that $E[|e_f(k|n)|^2] = E[|e_b(k|n)|^2]$ and that $K_n^f(k) = K_n^{b*}(k)$.

To evaluate (3), we determine the forward and backward prediction errors of an n-tap transversal filter. This is based on the assumption that the expected forward and backward prediction errors of the lattice filter at stage k are statistically equal to those of a k-step transversal filter.

3. OPTIMAL PARCOR COEFFICIENT

The signal model used here is that of a complex linear chirp FM in white Gaussian noise. The input signal x(k) is given by

$$x(k) = s(k) + n(k) \quad (4)$$

where n(k) is a zero mean white Gaussian noise signal and s(k) is a sinusoidal linear FM signal defined as

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$$s(k) = \sqrt{P_s}\, e^{j(k\omega + \frac{k^2\Psi}{2} + \Phi)} \quad (5)$$

where ω is the base frequency, Ψ is the chirp rate, Φ is a random phase that is assumed to be uniformly distributed between $-\pi$ and π, P_s is the signal power, and P_n is the noise power.

The characteristics of the SG Lattice filter can be derived by assuming that the backward and forward prediction errors at stage n are statistically equal to the respective backward and forward prediction errors of an n-stage linear predictor. The optimal PARCOR coefficient can be obtained by assuming that the backward and forward errors have attained their optimal values. This can be shown (Figure 2) to be

$$K_n^{opt}(k) = a^k b \frac{\rho}{1 + n\rho} \quad (6)$$

where $a = e^{j(n+1)\Psi}$, $b = e^{j[(n+1)\omega - \frac{(n+1)^2\Psi}{2}]}$, and $\rho = \frac{P_s}{P_n}$.

The update algorithm of the SG lattice algorithm does not permit the reflection coefficients to attain their optimal values for the non-stationary chirp FM signal. Instead, the "shadowing" effect is observed. Simulations with complex FM signals reveal that the actual reflection coefficients at each iteration contain two frequencies – the fundamental frequency at the start of the sweep and the swept frequency at that iteration. The next section will derive a first-order approximation of the reflection coefficient characteristics.

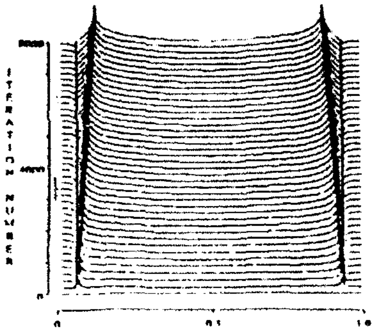

Figure 2 Transfer Function of the Expected Value of the Optimal PARCOR Coefficients

4. SINGLE STAGE PARCOR COEFFICIENT

The reflection coefficient update equation is given by

$$K_n(k+1) = [1 - \beta|e_b(k-1|n-1)|^2]K_n(k) + \beta e_f(k|n-1)e_b^*(k-1|n-1) \quad (7)$$

where β is the forgetting factor (equivalent to μ in the LMS algorithm). For convenience, we shall drop the subscripts in n and n-1. Taking expectation on both sides, the iterated solution of (7) is given by

$$E[K(k+1)] = K(0)E[\prod_{r=1}^{k}(1 - \beta|e_b(r-1)|^2)] + \beta E\left[\sum_{r=0}^{k-1} e_f(k-r)e_b^*(k-r-1)\prod_{s=1}^{r}[1 - \beta|e_b(k-s)|^2]\right] \quad (8)$$

Assuming that $K(0) = 0$, and neglecting terms in β^2 and higher, (8) can be evaluated to be

$$E[K(k+1)] = \frac{\beta b P_s}{1 + \rho n}\,\frac{a - a^{k+1}}{1 - a} \quad (9)$$

Due to the summation in (8), which turns out to be a geometric series, only the fundamental term (a) and the term at the k+1st iteration (a^{k+1}) remain. This serves to confirm the observed simulation results, as shown in Figures 3 and 4.

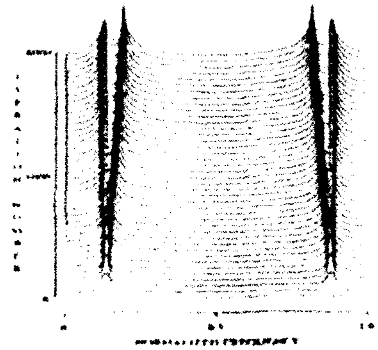

Figure 3 Transfer Function of the Expected PARCOR Coefficients from Simulation

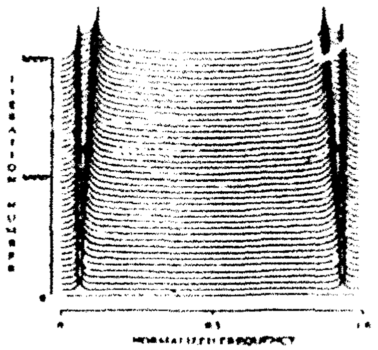

Figure 4 Transfer Function of the Expected PARCOR Coefficients from Single-Stage Model

5. SINGLE STAGE RECOVERY ERROR

The recovery error in this situation measures the amount of residual signal left in the forward error. We shall derive a simplistic single stage recovery error expression here. The recovery error is defined as

$$\eta(k|n) = e_f(k|n) - n(k) \quad (10)$$

Defining

$$\eta_o(k|n) = e_f(k|n-1) - K_n^o(k)e_b(k-1|n-1) - n(k) \quad (11)$$

we can rearrange (10) as

$$\eta(k|n) - \eta_o(k|n) = -\Gamma_n(k)e_b(k-1|n-1) \quad (12)$$

where

$$\Gamma_n(k) = K_n(k) - K_n^o(k)$$

We wish to determine the quantity $E[|\eta(k|n) - \eta_o(k|n)|^2]$. Following the methodology and assumptions of [6], the above quantity can be found to be

$$E[|\eta(k|n) - \eta_o(k|n)|^2] = \left|\frac{a-1}{M-a}\right|^2 \frac{\varrho^2 P_n}{[1+\varrho n][1+\varrho(n-1)]} + \frac{\beta P_n^2}{2}\frac{1+\varrho(n+1)}{1+\varrho(n-1)} \tag{13}$$

where

$$M = \frac{[1-\beta P_n][1+\varrho n] - \varrho}{1+\varrho(n-1)}$$

6. TWO-STAGE ANALYSIS (NOISELESS)

This section presents a two-stage analysis of the SG lattice filter assuming that the input contains no noise. The next section will analyze the case with input noise. For a two-stage model, the input signal is given by

$$e_f(k|0) = e_b(k|0) = \sqrt{P_s}\, e^{j(k\omega + \frac{k^2\Psi}{2} + \Phi)} \tag{14}$$

Letting $a' = e^{j\Psi}$ and $b' = e^{j\omega}$, we get

$$e_f(k-r|0)e_b^*(k-r-1|0) = P_s b' a'^{-1/2} a'^{(k-r)} \tag{15}$$

The reflection coefficient update formula is given by

$$K_1(k+1) = [1-\beta P_s]K_1(k) + \beta e_f(k|0)e_b^*(k-1|0) \tag{16}$$

Let $q = 1-\beta P_s$. Then from (15), $K_1(k+1)$ can be evaluated to be

$$K_1(k+1) = \beta P_s b' a'^k a'^{-1/2} \frac{a'^{(k+1)} - q^k a'^k}{a' - q} \tag{17}$$

The plot of K_1 verses iteration number is shown in Figure 7. The simulation and analytical model results are identical in this case.

From (17) we see that $K_1(k+1)$ has a transient and steady state component. The transient component is dependent on $1-\beta P_s$. This gives a convergence time constant of

$$\tau_1 = \frac{1}{\beta P_s} \tag{18}$$

When the chirp rate is zero, the term $\frac{\beta P_s}{a' - q}$ approaches unity. This gives $K_1(k+1)$ its maximum magnitude. With increasing chirp rate this term reduces $K_1(k+1)$ in the manner shown in Figure 5, where $\frac{\beta P_s}{a' - q}$ is graphed as a function of chirp rate for different β's.

To derive the second reflection coefficient $K_2(k+1)$, we shall approximate $K_1(k+1)$ by its steady-state value

$$K_1(k+1) \cong \beta P_s b' a'^k a'^{-1/2} \frac{a'^{(k+1)}}{a' - q} \tag{19}$$

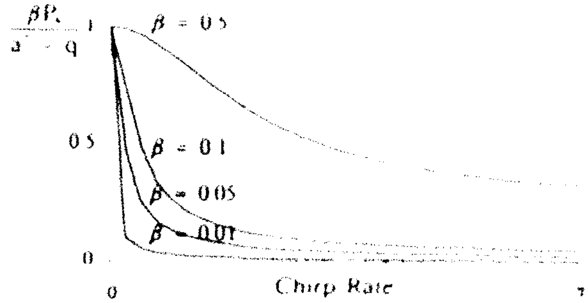

Figure 5 Plot of $\frac{\beta P_s}{a' - q}$ for different β and chirp rates

The forward and the backward prediction errors of the first stage are derived from the zeroth stage by the following iterative equations

$$e_f(k|1) = e_f(k|0) - K_1(k)e_b(k-1|0)$$
$$e_b(k|1) = e_b(k-1|0) - K_1^* e_f(k|0) \tag{20}$$

Defining $1 - \beta|e_f(k|1)|^2 = q_1$, the second reflection coefficient can be written as

$$K_2(k+1) = \beta\sum_{r=0}^{k-2} e_f(k-r|1)e_b^*(k-r-1|1)q_1 \tag{21}$$

Using the iteration of (20), (21) can be evaluated to be

$$K_2(k+1) = I_1 - I_2 - I_3 + I_4 \tag{22}$$

where

$$I_1 = \beta P_s b'^2 \frac{a'^{2k} - q_1^{k-1}a'^2}{a'^2 - q_1} \tag{23}$$

$$I_2 = I_3 = \frac{\beta^2 P_s^2 b'^2}{a-q}\left[\frac{a'^{2k} - q_1^{k-1}a'^2}{a'^2 - q_1} - \frac{q^{k-1}a'^{k+1} - q_1^{k-1}a'^2}{qa - q_1}\right] \tag{24}$$

$$I_4 = \frac{\beta^3 P_s^3 b'^2}{a-q}\left[\frac{a'^{2k} - q_1^{k-1}a'^2}{a'^2\ q_1} - \frac{q^{k-1}a'^{k+1} - q_1^{k-1}a'^2}{qa - q_1} - \frac{q^k a^k - q_1^k qa'}{qa - q_1} + \frac{q^{2k-1}a' - q_1^{k-1}qa'}{q^2 - q_1}\right]$$

Examining I_1–I_4 we see that the convergence of $K_2(k+1)$ is dependent both on q and q_1. The time constant related to q is given by (18). The time constant related to q_1 is given by

$$\tau_2 = \frac{1}{\beta P_s\left[1 - \frac{\beta P_s(2\cos\Psi - 2q - \beta P_s)}{q^2 - 2q\cos\Psi + 1}\right]} \tag{26}$$

Figure 6 shows a plot of K_2 versus iteration number. Note that the convergence rate of the second stage is slower than that of the first. The steady-state value of $K_2(k+1)$ can be derived to be

$$\lim_{k \to \infty} K_2(k+1) = \frac{\beta P_s b^{*2} a^{*2k}}{a^{*2} - q} \left[1 - \frac{2\beta P_s}{a^* - q} + \frac{\beta^2 P_s^2}{(a^* - q)^2} \right] \quad (27)$$

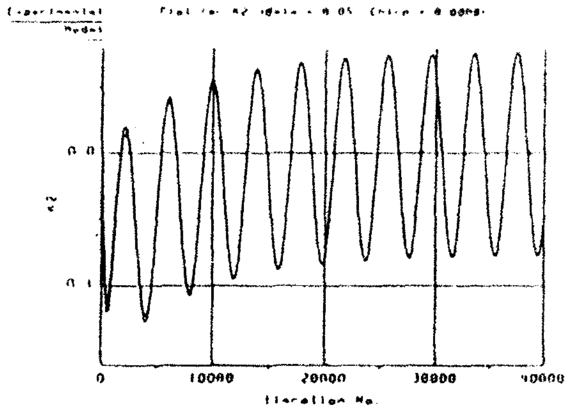

Figure 6 Real Component of the Second PARCOR Coefficient (K2)

7. FIRST STAGE ANALYSIS WITH NOISE

The general iterated solution for the first reflection coefficient is given by

$$K_1(k+1) = \beta \sum_{r=0}^{k-1} e_f(k-r|0) e_b^*(k-r-1|0) \cdot \prod_{s=1}^{r} \left\{ 1 - \beta e_b^2(k-s|0) \right\} \quad (28)$$

The noise terms in $1 - \beta e_b^2(k-s|0)$ for s=1 to s=r-1 are independent of each other and of $e_f(k-r|0)e_b^2(k-r-1)\left\{ 1 - \beta e_b^2(k-s|0)\right\}$. Thus $E[K_1(k+1)]$ can be written as

$$E[K_1(k+1)] = \beta \sum_{r=0}^{k-1} E\left[e_f(k-r|0) e_b^*(k-r-1|0) \cdot \left\{ 1 - \beta e_b^2(k-r|0) \right\} \right] \prod_{s=1}^{r-1} \left\{ 1 - \beta E[|e_b(k-s|0)|^2] \right\} \quad (29)$$

Letting $q_n = \left\{ 1 - \beta E[e_b^2(k-s|0)\right\} = 1 - \beta(P_s + P_n)$, the summation can be evaluated to be

$$E[K_1(k+1)] = \beta P_s b' a'^{-1/2} \frac{a'^{(k+1)} - q_n^k a'}{a' - q_n} \cdot \frac{1 - \beta(P_s + 2P_n)}{1 - \beta(P_s + P_n)} \quad (30)$$

We see that the reflection coefficient depends upon the signal and noise by the factor $\frac{\beta P_s}{a' - q_n} \cdot \frac{1 - \beta(P_s + 2P_n)}{1 - \beta(P_s + P_n)}$ and the convergence constant is now given by

$$\tau_{n_1} = \frac{1}{\beta(P_s + P_n)} \quad (31)$$

Figure 7 shows the plot of (30) superimposed on the actual simulation. The second reflection coefficient iteration formula has also been derived, but shall not be presented due to space considerations.

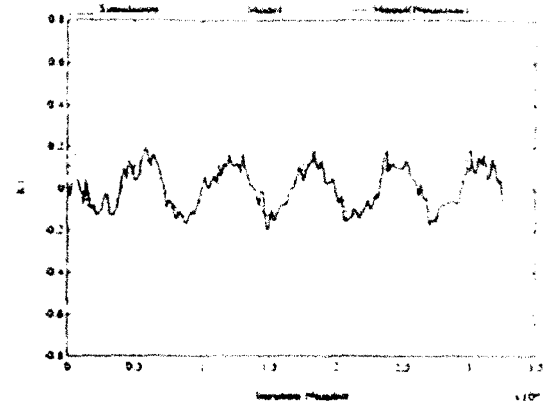

Figure 7 Real Component of the First PARCOR Coefficient (K1) with noise

8. CONCLUSION

In this paper, we have studied the behavior of the PARCOR coefficients of the SG Lattice Filter for a complex linear chirp FM in white noise. Independent terms which define the tracking and convergence rates are derived.

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